Mark Scheme (Results)

Summer 2024

Pearson Edexcel International Advanced Level
In Further Pure Mathematics (WFM01) Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. Edexcel Mathematics mark schemes use the following types of marks:
   - 'M' marks
     - These are marks given for a correct method or an attempt at a correct method.
   - 'A' marks
     - These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. e.g. M0 A1 is impossible.
   - 'B' marks
     - These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph).
   - A and B marks may be f.t. – follow through – marks.

   Marks should not be subdivided

3. General Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes:

   - bod means benefit of doubt
   - ft means follow through
     - the symbol $\sqrt{}$ will be used for correct ft
   - cao means correct answer only
   - cso means correct solution only, i.e. there must be no errors in this part of the question to obtain this mark
   - isw means ignore subsequent working
   - awrt means answers which round to
   - SC means special case
   - oe means or equivalent (and appropriate)
   - dep means dependent
   - indep means independent
   - dp means decimal places
   - sf means significant figures
   - * means the answer is printed on the question paper
   - ⎿ means the second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

- Factorisation
    - o $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = ...$
    - o $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = ...$
- Formula
    - o Attempt to use the correct formula (with values for *a*, *b* and *c*).
- Completing the square
    - o Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to $x = ...$

Method marks for differentiation and integration:

- Differentiation
    - o Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)
- Integration
    - o Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first. Normal marking procedure is as follows:

- Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.
- Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

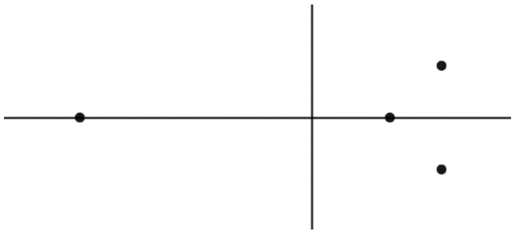
Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Notes</th><th>Marks</th></tr>
<tr><td>1(i)</td><td colspan="2">$\mathbf{A}=\begin{pmatrix}3k & 4k-1\\ 2 & 6\end{pmatrix}$</td><td></td></tr>
<tr><td>(a)</td><td colspan="2">$3k\times6-2(4k-1)=0\Rightarrow k=...$
Forms det A = 0 and solves for k.
The " = 0" can be implied by a solution for k.
Award for $3k\times6-2(4k-1)=0\Rightarrow k=...$
If LHS is only seen expanded 2 terms of $18k-8k+2$ must be correct (implied by $10k$)
May use $ad=bc$ and condone $\det\mathbf{A}=bc-ad=0$ but clear use of $ad+bc$ is M0</td><td>M1</td></tr>
<tr><td></td><td>$(10k+2=0\Rightarrow k=)-\frac{1}{5}$ or -0.2</td><td>A1: Correct value. Accept $-\frac{2}{10}$</td><td>A1</td></tr>
<tr><td colspan="4" align="right">(2)</td></tr>
<tr><td>(b)</td><td colspan="2">$(\mathbf{A}^{-1}=)\ \frac{1}{10k+2}\begin{pmatrix}6 & 1-4k\\ -2 & 3k\end{pmatrix}$ or $\begin{pmatrix}\frac{6}{10k+2} & \frac{1-4k}{10k+2}\\ \frac{-2}{10k+2} & \frac{3k}{10k+2}\end{pmatrix}$ or e.g., $\begin{pmatrix}\frac{3}{5k+1} & \frac{1-4k}{10k+2}\\ \frac{-1}{5k+1} & \frac{3k}{10k+2}\end{pmatrix}$

M1: for $...\begin{pmatrix}6 & 1-4k\\ -2 & 3k\end{pmatrix}$ Ignore any multiplier and accept without one and condone if this matrix is labelled as $\mathbf{A}^{-1}$. Allow unsimplified e.g., $...\begin{pmatrix}6 & -(4k-1)\\ -2 & 3k\end{pmatrix}$
Allow if determinant incorporated provided it is clear that the elements of Adj(A) are correct
A1ft: $\frac{1}{\text{"}10k+2\text{"}}\begin{pmatrix}6 & 1-4k\\ -2 & 3k\end{pmatrix}$ Fully correct inverse ft their determinant in form $ak+b\quad a,b\neq0$ and simplified but if determinant incorporated there is no requirement to write e.g., $\frac{6}{10k+2}$ as $\frac{3}{5k+1}$. Allow different brackets e.g., [...], {...} but $|...|$ is M0 if followed by an attempt at det(Adj(A)). Allow if "×" is between fraction and matrix and allow fraction to appear on the right of the matrix. Isw when a correct answer is seen but this mark is not available if they substitute a value of k into the determinant and/or matrix.</td><td>M1
A1ft</td></tr>
<tr><td colspan="4" align="right">(2)</td></tr>
<tr><td>(ii)(a)</td><td>$p=q=-2$ or $(\mathbf{B}=)\begin{pmatrix}-2 & 0\\ 0 & -2\end{pmatrix}$</td><td>Both values identified or correct matrix (any or no bracket).
Allow "Both are -2" or "-2, -2"</td><td>B1</td></tr>
<tr><td>(b)</td><td>$p=-1\quad q=1$ or $(\mathbf{B}=)\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}$</td><td>Both values identified or correct matrix (any or no bracket). Allow "-1, +1" (Mark in order presented). No trig expressions.</td><td>B1</td></tr>
<tr><td colspan="4" align="right">(2)</td></tr>
<tr><td colspan="4" align="right">Total 6</td></tr>
</table>

Question Number	Scheme	Notes	Marks
2	$\mathrm{f}(z)=z^3-13z^2+59z+p$		
(a)	$[\mathrm{f}(3)=]3^3-13(3)^2+59(3)+p$ or e.g., $27-117+177+p$ or $z-3 \overline{)z^3-13z^2+59z+p}$ giving $z^2-10z+29$: z^3-3z^2 $-10z^2+59z$ $-10z^2+30z$ $29z+p$ $29z-87$ 0	Attempts $\mathrm{f}(3)$. Must see more than just $87+p$ Allow one slip (e.g., a miscopy of one coefficient, or one incorrect value/sign if expression just given as 27 – 117 +177 + p) **Alternatively** long divides by $z-3$ obtaining a 3TQ with two terms of $z^2-10z+29$ correct. Could use synthetic division. An attempt at equating coefficients/factorising requires 2 correct values for the *a*, *b* and *c* of az^2+bz+c	**M1**
	$\mathrm{f}(3)=0 \Rightarrow p=-87$ *	Obtains "$p=-87$" only with no errors but condone work in *x* "=0" must have been seen before $p=-87$ if f(3) attempted but allow just $p=-87$ following a full and correct attempt via division/equating coefficients etc with no errors.	**A1* (shown as B1 on ePen)**
			(2)
(b)	Allow equivalent work in *x*. Allow use of a calculator to solve a **quadratic**. Solutions that just follow $z^3-13z^2+59z-87=0$ score no marks. There are no marks if $z^2-10z+29$ has clearly been produced by using $(z-(5+2\mathrm{i}))(z-(5-2\mathrm{i}))$		
	$(z^3-13z^2+59z-87)\div(z-3)$ $=\ldots\ [z^2-10z+29]$	M1: Uses $z\pm3$ with f(*z*) (not their f(*z*)) to obtain a 3TQ expression with evidence of any appropriate method including inspection (must be evidence of use of $z\pm3$) or equating coefficients. Ignore any remainder if long division is used and may see $z^2-16z+107\ (r(-408))$ if $z+3$ used. Must be seen or referred to in (b) A1: Correct quadratic	**M1** **A1**
	$z=\dfrac{-(-10)\pm\sqrt{(-10)^2-(4)(1)(29)}}{2(1)}$ or $(z-5)^2-25+29=0 \Rightarrow z=5\pm\sqrt{-4}$	Solves their 3TQ arising from using $(z-3)$ only as a factor (usual rules but allow if one correct root if calculator used on their quadratic) If a sum/product of roots method is used on their 3TQ$\left(\text{i.e., } 2a=-(\text{"}-10\text{"}),\ a^2+b^2=\text{"}29\text{"}\right)$ it must be complete and condone only sign errors. Do not allow just $5\pm2\mathrm{i}$ following an incorrect quadratic **Requires previous M mark.**	**dM1**
	$\left(z=\dfrac{10\pm\sqrt{-16}}{2}=\right)\ 5\pm2\mathrm{i}$	$5\pm2\mathrm{i}$ or 5 + 2i, 5 – 2i only. Not $5\pm2\sqrt{-1}$ Accept ±2i+5	**A1**
			(4)

Question Number	Scheme	Notes	Marks
2(c)	 Look for this arrangement if correct but note potential ft	Correct diagram ft their $a \pm b\mathrm{i}$ $(a,\ b \neq 0)$ Diagram should be roughly symmetrical in the real axis. The point on the negative *x*-axis should be further from the origin than the point on the positive *x*-axis but ignore any other scaling issues – just look for the $a \pm b\mathrm{i}$ points to be placed in the correct quadrants, roughly aligned vertically and placed correctly relative to the given point that is on the same side of the *y*-axis. Points/axes may be unlabelled or mislabelled. If vectors/lines are used the end points must satisfy the conditions above.	**B1ft**
			(1)
(d)	$2\left(\sqrt{("5"-(-9))^2+"2"^2}+\sqrt{("5"-3)^2+"2"^2}\right)$	A correct numerical expression for the perimeter ft their $a \neq 0$ or 3 or -9 and $b \neq 0$ This mark requires working with points that would form a convex or concave kite where the *x*-axis is a line of symmetry. Working must be seen if $a \pm b\mathrm{i}$ incorrect but allow just $4\sqrt{5}+4\sqrt{17}$ oe from using $-5 \pm 2\mathrm{i}$	**M1**
	$\left[=2\left(\sqrt{14^2+2^2}+\sqrt{2^2+2^2}\right)=2\left(\sqrt{200}+\sqrt{8}\right)\right]$ $=24\sqrt{2}$	$24\sqrt{2}$ or any simplified equivalent e.g., $12\sqrt{8}$ or $2\sqrt{288}$ but not $\sqrt{1152}$. Correct answer scores both marks and allow M1 A0 for just $\sqrt{1152}$	**A1**
			(2)
			Total 9

Question Number	Scheme	Notes	Marks
3	$\mathrm{f}(x)=x^3-5\sqrt{x}-4x+7$		
(a)	f(0.25)=3.515625, $\frac{225}{64}$, $3\frac{33}{64}$ f(1) = −1	Attempts both f(0.25) and f(1) with one correct allowing awrt 3.52 for f(0.25)	**M1**
	examples: “1” refers to “–1” with sign corrected $\frac{\alpha-0.25}{"3.515625"}=\frac{1-\alpha}{"1"}\Rightarrow\alpha=...$ $\frac{\alpha-0.25}{"\frac{225}{64}"}=\frac{1-\alpha}{"1"}\Rightarrow\alpha=...$ $\frac{\alpha-0.25}{"3.515625"}=\frac{1-0.25}{"3.515625"+"1"}\Rightarrow\alpha=...$ $\frac{1-\alpha}{"1"}=\frac{1-0.25}{"3.515625"+"1"}\Rightarrow\alpha=...$ [$\alpha-0.25=3.515625-3.515625\alpha$ $4.515625\alpha=3.765625$]	Forms an equation in α that is correct for their values and solves for α. Can use *x* etc. Allow e.g., “f(0.25)” and “-f(1)” in this equation provided values for these are seen. Any modulus signs must be applied and f(0.25) and f(1) must have had different signs. Can be implied by just awrt 0.83 or $\frac{241}{289}$ but otherwise a correct equation for their f(0.25) and f(1) must be seen but allow use of $\frac{a\mathrm{f}(b)-b\mathrm{f}(a)}{\mathrm{f}(b)-\mathrm{f}(a)}\Rightarrow\frac{1("3.515625")-0.25("-1")}{"3.515625"-("-1")}$ or a correct partially processed equivalent and only allow formula followed by value if values for *a*, *b*, f(*a*) and f(*b*) are seen. If e.g., *A* is used for $\alpha-0.25$ then must see $A+0.25$ later. Note that sight of 1.2981... or $\frac{209}{161}$ usually indicates a sign error.	**M1**
	$\alpha=0.834$	awrt 0.834 (0.8339100346...) Must be decimal. Ignore labelling and just look for this value. [Note: actual root is 0.767843...]	**A1**
			(3)
Alt for last 2 marks (straight line equation)	e.g., $y=\frac{"3.515625"-"(-1)"}{0.25-1}x+c$ $(1,"-1")\Rightarrow-1=-6.0208\dot{3}+c$ $\Rightarrow c=5.0208\dot{3}$ $y=0\Rightarrow\alpha=\frac{-5.0208\dot{3}}{-6.0208\dot{3}}=0.834$	M1: Any full method to find the equation of the line between (0.25, “3.515625”) and (1, “–1”) and then uses *y* = 0 to find a value for α. Condone errors finding *c* and α but the initial equation should be correct for their f(0.25) and f(1) and the *x* and *y* coordinates should always be correctly placed. A1: awrt 0.834	**M1** **A1**
(b)	$[\mathrm{f}'(x)=]\ 3x^2-\frac{5}{2}x^{-\frac{1}{2}}-4$	M1: 2 correctly differentiated terms (this includes $7\to0$) Allow unsimplified e.g., $3\times x^{3-1}$ A1: Fully correct simplified derivative	**M1** **A1**
			(2)
(c)	$x_1=1.75-\frac{1.75^3-5\sqrt{1.75}-4(1.75)+7}{"3(1.75)^2-2.5(1.75)^{-0.5}-4"}$ $\left[=1.75-\frac{-1.255003278...}{3.297677635...}=1.75+0.38057...\right]$	Uses a correct Newton-Raphson formula with $x_0=1.75$ and their $\mathrm{f}'(x)$ to obtain a numerical expression for x_1 but implied by awrt 2.13 (2.13057185). Working must be seen if x_1 is wrong – allow "$x_0=1.75,\ x_1=x_0-\frac{\mathrm{f}(x_0)}{\mathrm{f}'(x_0)}=...$"or $1.75-\frac{\mathrm{f}(1.75)}{\mathrm{f}'(1.75)}=...$"	**M1**
	$x_1=2.13057185...\Rightarrow\beta=2.131$	awrt 2.131 (ignore labelling and just look for this value). Ignore further iterations. [Note: actual root is 2.011276...]	**A1**
			(2)
			Total 7

Question Number	Scheme	Notes	Marks
4	If z is **restated** incorrectly, e.g., "$z = 3+4\text{i}$" is seen allow a maximum of M1dM1A0 B0M1A1M1A0		
(a)	$z^2 - 3 = (-3+4\text{i})(-3+4\text{i}) - 3$ $= 9 - 24\text{i} - 16 - 3$ $= -10 - 24\text{i}$	Substitutes $z = -3+4\text{i}$ into $z^2 - 3$, expands and reaches $a + b\text{i}$ $(a,\ b \neq 0)$ Implied by $-10-24\text{i}$ seen and condone misapplication of the modulus e.g., using $a+b\text{i}$ from $\lvert -a - b\text{i}\rvert$	**M1**
	$\lvert z^2 - 3\rvert = \sqrt{"10"^2 + "24"^2}$	Correct expression for modulus of their $a + b\text{i}$ $(a,\ b \neq 0)$ Allow with no working for the modulus provided answer correct for their $a + b\text{i}$ **Requires previous M mark.**	**dM1**
	26	26 only from correct work. e.g., $\lvert -10+24\text{i}\rvert = 26$ is A0 Answer only or without $-10-24\text{i}$ is no marks.	**A1**
			(3)
(b)	$(z = -3+4\text{i} \Rightarrow)\quad z^* = -3-4\text{i}$	Correct conjugate. Can be implied	**B1**
	$\frac{50}{z^*} = \frac{50}{-3-4\text{i}} \times \frac{-3+4\text{i}}{-3+4\text{i}}\ \left[= 50 \times \frac{-3+4\text{i}}{25}\right]$ or $\frac{1}{z^*} = \frac{1}{-3-4\text{i}} \times \frac{-3+4\text{i}}{-3+4\text{i}}\ \left[= \frac{-3+4\text{i}}{25}\right]$	A correct multiplier seen that would make the denominator real for $\frac{50}{z^*}$ or $\frac{1}{z^*}$ where $z^* = \pm 3 \pm 4\text{i}$ (**except** $-3+4\text{i}$). If the multiplier is not seen must see something better than $50 \times \frac{-3+4\text{i}}{25}$ or $\frac{-3+4\text{i}}{25}$ or –6+8i e.g., $\frac{50}{z^*} = \frac{50(-3+4\text{i})}{9+16}$	**M1**
	$\frac{50}{z^*} = 2(-3+4\text{i})$ or $2z$	Obtains $2(-3+4\text{i})$ or $2z$ Just $-6+8\text{i}$ is insufficient Allow $k = 2$ provided "$= kz$" or "$= k(-3+4\text{i})$" is seen	**A1**
			(3)
Using Result	May see : $\frac{50}{-3-4i} = k(-3+4\text{i}) \Rightarrow \frac{50}{9+16} = k \Rightarrow k = 2$ B1:Correct z^* M1: $\frac{50}{9+16} = k$ or better after multiplication A1*: $k = 2$		
Alt **Using** $\frac{1}{z^*} = \frac{z}{\lvert z\rvert^2}$	$\frac{1}{z^*} = \frac{z}{\lvert z\rvert^2}$ oe e.g., $z^* z = \lvert z\rvert^2$	States or uses $\frac{1}{z^*} = \frac{z}{\lvert z\rvert^2}$ oe	**B1**
	$\frac{c}{z^*} = \frac{cz}{\lvert z\rvert^2}$, $\lvert z\rvert = \sqrt{3^2 + 4^2} = ...$	Expresses $\frac{c}{z^*}$ as $\frac{cz}{\lvert z\rvert^2}$ and attempts $\lvert z\rvert$ or $\lvert z\rvert^2$ where $c = 1$ or 50	**M1**
	$\frac{50}{z^*} = \frac{50z}{25} = 2z$	Correctly finds $2z$ Allow $k = 2$ provided "$= kz$" or "$= k(-3+4\text{i})$" is seen	**A1**

Question Number	Scheme	Notes	Marks
4(c)	$\arctan\left(\pm\frac{4}{3}\right) = \pm 0.927...(53.13^\circ)$ or $\arctan\left(\pm\frac{3}{4}\right) = \pm 0.643...(36.86^\circ)$ May see equivalent trig in which case the hypotenuse should be correct	Finds a relevant angle which could be in degrees correct to 2sf so accept awrt $\pm 0.93\ (53^\circ)$ or $\pm 0.64\ (37^\circ)$ If neither value is seen allow implication from the work May see e.g., $\tan^{-1}\left(\pm\frac{8}{6}\right) = ...$ M0 if arg 2*z* replaced with 2 arg *z*	**M1**
	$\left[\theta = \pi - 0.927295... \quad \theta = \frac{\pi}{2} + 0.643501...\right]$ $\theta = 2.21$	**Final answer** of awrt 2.21 – **do not isw**. (n.b. $\theta = 2.214297436...$) Final answer of e.g., "$\pi - 0.927$" is A0 Answer only scores both marks. Answer only in degrees (awrt 127°) is M1A0	**A1**
Note: allow access to both marks even if *k* in part (b) was incorrect			**(2)**
			Total 8

Question Number	Scheme	Notes	Marks
5	$5x^2-4x+2=0$		
	Solutions that rely on solving the given quadratic/finding values for p and q are likely to score a maximum of 0010 11010 if the relevant work is seen		
(a)(i)	$\frac{1}{p}\times\frac{1}{q}$ or $\frac{1}{pq}=\frac{2}{5}\Rightarrow pq=\frac{5}{2}$ *	Shows product of roots $=\frac{2}{5}$ followed by $pq=\frac{5}{2}$ Minimum as shown. Allow e.g., $qp=2.5$ Note that $\frac{1}{pq}=\frac{1}{\frac{2}{5}}\Rightarrow pq=\frac{5}{2}$ is B0 No clearly incorrect work/statements.	**B1***
	May see: $\left(x-\frac{1}{p}\right)\left(x-\frac{1}{q}\right)=x^2-\left(\frac{1}{p}+\frac{1}{q}\right)x+\frac{1}{pq}=x^2-\frac{4}{5}x+\frac{2}{5}\Rightarrow\frac{1}{pq}=\frac{2}{5}\Rightarrow pq=\frac{5}{2}$ * Must not be any clearly incorrect work/statements.		
	Assuming result: $pq=\frac{5}{2}\Rightarrow\frac{1}{p}\times\frac{1}{q}=\frac{2}{5}$ requires conclusion e.g., "Hence true"		
(a)(ii) **May use work from (i)**	$\frac{1}{p}+\frac{1}{q}=-\frac{(-4)}{5}$	Uses sum of roots to achieve a correct equation in p and q	**M1**
	$\frac{1}{p}+\frac{1}{q}=\frac{p+q}{pq}$	States or uses $\frac{1}{p}+\frac{1}{q}=\frac{p+q}{pq}$	**M1**
	$\frac{p+q}{pq}=\frac{p+q}{\frac{5}{2}}=\frac{4}{5}\Rightarrow p+q=\frac{4}{5}\times\frac{5}{2}=2$	"$p+q=2$" from correct work. Allow "$2=q+p$"	**A1**
			(4)
Alt $x\to\frac{1}{z}$	$x\to\frac{1}{z}\Rightarrow 5\left(\frac{1}{z}\right)^2-4\left(\frac{1}{z}\right)+2=0$	Correctly replaces x with e.g., $\frac{1}{z}$ and allow $\frac{1}{x}$	**1st M1**
	$2z^2-4z+5=0$	Obtains a 3TQ in "z", "w" etc.	**2nd M1**
	$pq=\frac{5}{2}$	States $pq=\frac{5}{2}$ following correct work	**B1*** **1st mark**
	$p+q=2$	"$p+q=2$" from correct work	**A1**

Question Number	Scheme	Notes	Marks
5(b)	$\frac{p}{p^2+1}+\frac{q}{q^2+1}=\frac{pq^2+p+p^2q+q}{p^2q^2+p^2+q^2+1}$ $\frac{p}{p^2+1}\times\frac{q}{q^2+1}=\frac{pq}{p^2q^2+p^2+q^2+1}$ M1: For $p(q^2+1)+q(p^2+1)\to pq^2+p+p^2q+q$ or $(p^2+1)(q^2+1)\to p^2q^2+p^2+q^2+1$ Allow equivalents e.g., $pq(p+q)+p+q$ provided the initial expansion has been carried out A1: Both correct (expression for denominator seen correctly once) Do not accept pq^2 for $(pq)^2$ unless it is clearly recovered		**M1** **A1**
	sum $=\frac{pq(p+q)+p+q}{(pq)^2+(p+q)^2-2pq+1}=\frac{\frac{5}{2}\times 2+2}{\left(\frac{5}{2}\right)^2+2^2-2\times\frac{5}{2}+1}=\frac{7}{\frac{25}{4}}=...\left(\frac{28}{25}\text{ or } 1.12\right)$ product $=\frac{pq}{(pq)^2+(p+q)^2-2pq+1}=\frac{\frac{5}{2}}{\left(\frac{5}{2}\right)^2+2^2-2\left(\frac{5}{2}\right)+1}=\frac{\frac{5}{2}}{\frac{25}{4}}...\left(\frac{2}{5}\text{ or } 0.4\right)$ Obtains a value for either the new sum or new product using $\underline{pq=\frac{5}{2}}$ and a value for $p+q$ which could be their answer from part (a)(ii) and may have been stated as e.g., $\frac{1}{p}+\frac{1}{q}$ or it could be inconsistent with their answer to (a)(ii). May be slips. At least one of their expressions must have included both pq and $p+q$ and have been completely in terms of pq and $p+q$ including at least one use of $p^2+q^2=(p+q)^2-2pq$. Accept just sum $=\frac{28}{25}$ or product $=\frac{2}{5}$ if there is no clearly incorrect work otherwise some evidence of all of the above conditions and not just values must be seen. **Requires previous M mark.**		**dM1**
	Note that for the numerator of the sum it is possible to use $pq^2+p+p^2q+q=p+q+(p+q)(p^2+q^2)-(p^3+q^3)=p+q+(p+q)\left((p+q)^2-2pq\right)-\left((p+q)^3-3pq(p+q)\right)$ in which case both $p^2+q^2=(p+q)^2-2pq$ and $p^3+q^3=(p+q)^3-3pq(p+q)$ must be used		
	The above work may be embedded within $x^2\pm(\text{sum})x\pm\text{product}$		
	$x^2-\frac{28}{25}x+\frac{2}{5}$	Applies $x^2-(\text{sum})x+\text{product}$ correctly for their stated **values** for new sum and product. Not dependent.	**M1**
	$25x^2-28x+10=0$	Correct quadratic (or integer multiple) with "= 0" Allow a different variable e.g., z for x Allow e.g., $a=25,\ b=-28,\ c=10$ provided $ax^2+bx+c=0$ is seen otherwise score M1A0	**A1**
			(5)
			Total 9

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Notes</th><th>Marks</th></tr>
<tr><td>6(a)</td><td colspan="2">$\begin{pmatrix}1 & r\\0 & 2\end{pmatrix}^n = \begin{pmatrix}1 & (2^n-1)r\\0 & 2^n\end{pmatrix}$</td><td></td></tr>
<tr><td></td><td colspan="2">Evaluates LHS & RHS for $n = 1$. LHS & RHS indicated (or "true" seen) if not equated
$(\text{LHS}=)\begin{pmatrix}1 & r\\0 & 2\end{pmatrix}^1$ or $\begin{pmatrix}1 & r\\0 & 2\end{pmatrix} = \begin{pmatrix}1 & (2^1-1)r\\0 & 2^1\end{pmatrix}$ or $\begin{pmatrix}1 & (2-1)r\\0 & 2\end{pmatrix}(=\text{RHS})$</td><td>B1</td></tr>
<tr><td></td><td colspan="2">Assume true for $n = k$, i.e., $\begin{pmatrix}1 & r\\0 & 2\end{pmatrix}^k = \begin{pmatrix}1 & (2^k-1)r\\0 & 2^k\end{pmatrix}$</td><td></td></tr>
<tr><td></td><td>$\begin{pmatrix}1 & r\\0 & 2\end{pmatrix}^{k+1} = \begin{pmatrix}1 & r\\0 & 2\end{pmatrix}\begin{pmatrix}1 & (2^k-1)r\\0 & 2^k\end{pmatrix}$</td><td>Uses $n = k$ result to form expression for $\begin{pmatrix}1 & r\\0 & 2\end{pmatrix}^{k+1}$
Implied by 3 correct elements if they immediately multiply provided the result is not just the "given" answer and allow this to be the intermediate step</td><td>M1</td></tr>
<tr><td></td><td>$= \begin{pmatrix}1 & (2^k-1)r+2^k r\\0 & 2(2^k)\end{pmatrix} = \begin{pmatrix}1 & (2^{k+1}-1)r\\0 & 2^{k+1}\end{pmatrix}$</td><td>Correct result with intermediate step that involves the top right element and no errors seen in the algebra. Allow "meet in the middle" proofs.
Only allow $(2^{k+1}-1)r$ written as $r(2^{k+1}-1)$ or $(-1+2^{k+1})r$ or $r(-1+2^{k+1})$. No $2(2^k)$s for 2^{k+1}</td><td>A1</td></tr>
<tr><td></td><td colspan="2">Alternatively: $\begin{pmatrix}1 & r\\0 & 2\end{pmatrix}^{k+1} = \begin{pmatrix}1 & (2^k-1)r\\0 & 2^k\end{pmatrix}\begin{pmatrix}1 & r\\0 & 2\end{pmatrix} = \begin{pmatrix}1 & r+2(2^k-1)r\\0 & 2(2^k)\end{pmatrix} = \begin{pmatrix}1 & (2^{k+1}-1)r\\0 & 2^{k+1}\end{pmatrix}$</td><td></td></tr>
<tr><td></td><td colspan="2">True for n =1, if true for n = k <u>then</u> true for n = k + 1, true for all (positive integers) n
Correct conclusion or narrative. Minimum in bold.
"Assume true for $n = k$ ... true for $n = k+1$" is sufficient for the "<u>then</u>"
The two previous marks are required and this mark can only follow B0 if the B mark was only withheld for insufficient working provided there was an attempt with $n = 1$. Ignore further verifications for $n = 2$ etc. Condone "for all $n \in \mathbb{Z}$" but not $n \in \mathbb{R}$
Condone work with n used for k.</td><td>A1</td></tr>
<tr><td colspan="4" align="right">(4)</td></tr>
<tr><td>(b)(i)</td><td>$\begin{pmatrix}1 & -2\\0 & 2\end{pmatrix}^4 = \begin{pmatrix}1 & (2^4-1)(-2)\\0 & 24\end{pmatrix} = \begin{pmatrix}1 & -30\\0 & 16\end{pmatrix}$</td><td>Correct matrix N. Could come from manual multiplication or calculator</td><td>B1</td></tr>
<tr><td>(ii)</td><td>$\mathbf{B} = \mathbf{NM} = \begin{pmatrix}1 & -30\\0 & 16\end{pmatrix}\begin{pmatrix}4 & 0\\0 & 5\end{pmatrix} = \ldots$</td><td>Attempts NM with their N. Must not be MN.
The N must have exactly three non-zero elements with 0 as the first element in the second row and their NM must have three elements correct for their matrices</td><td>M1</td></tr>
<tr><td></td><td>$\begin{pmatrix}4 & -150\\0 & 80\end{pmatrix}$</td><td>Correct matrix B</td><td>A1</td></tr>
<tr><td colspan="4" align="right">(3)</td></tr>
<tr><td>(c)</td><td>$\det \mathbf{B} = 4\times 80-(0\times(-150)) = 320$
area $S = \dfrac{720}{320}$</td><td>A correct non-zero value for the determinant of their B (no more than two zero elements) and divides this result into 720 to obtain a value for the area</td><td>M1</td></tr>
<tr><td></td><td>$\dfrac{9}{4}$ or $2\dfrac{1}{4}$ or 2.25</td><td>Correct area. Any exact equivalent. <u>Must follow a correct B.</u> Answer only is M1A1 if B correct.</td><td>A1</td></tr>
<tr><td colspan="4" align="right">(2)</td></tr>
<tr><td colspan="4" align="right">Total 9</td></tr>
</table>

Question Number	Scheme	Notes	Marks
7(a)	$\sum_{r=1}^{n}(12r^2+2r-3)=12\sum_{r=1}^{n}r^2+2\sum_{r=1}^{n}r-3\sum_{r=1}^{n}1$ $=12\times\frac{n}{6}(n+1)(2n+1)+2\times\frac{n}{2}(n+1)-3n$ $\left[=2n(n+1)(2n+1)+n(n+1)-3n\right]$	M1: Expands summation to at least 2 separate sums with one correct (could be implied), uses $\sum_{r=1}^{n}r^2=\frac{n}{6}(n+1)(2n+1)$ (allowing one of the following slips within the formula above: One of the 2 + signs seen as – or a missing first n) **and** replaces $\sum_{r=1}^{n}r$ with $\frac{n}{2}(n+1)$ **or** $\sum_{r=1}^{n}1$ with n Condone r used for n for the first three marks only. Allow $\sum$ for $\sum_{r=1}^{n}$ A1: Fully correct unsimplified expression	**M1** **A1**
	$\sum_{r=1}^{n}(12r^2+2r-3)=4n^3+6n^2+2n+n^2+n-3n=...$	Expands to a **cubic** and collects terms. Allow slips. **Requires previous M mark.**	**dM1**
	$4n^3+7n^2$	Correct expression from correct work Allow $A=4,\ B=7$ following $"=An^3+Bn^2"$	**A1**
			(4)
(b)	Full marks in (b) does not require full marks in (a)		
	$\sum_{r=1}^{2n}r^3=\frac{(2n)^2}{4}(2n+1)^2$	Attempts to use the sum of cubes formula with $2n$ Allow **one** of the following two slips: $2n^2$ for $(2n)^2$ Only one of the n's in the formula replaced by $2n$	**M1**
	$\sum_{r=1}^{2n}r^3-\sum_{r=1}^{n}(12r^2+2r-3)=$ $4n^4+4n^3+n^2-"4"n^3-"7"n^2\left[=270\right]$ $\left[\Rightarrow 4n^4-6n^2=270\right]$	Correct expanded quartic expression for $\sum_{r=1}^{2n}r^3-\sum_{r=1}^{n}(12r^2+2r-3)$ (ft their An^3+Bn^2) No requirement to collect terms but must be correct for their A and B if expression only seen with terms collected. If this is only seen as an equation it must be correct.	**A1ft**
	$4n^4-6n^2-270=0\Rightarrow$ $2n^4-3n^2-135=(2n^2+15)(n^2-9)=0$ $\Rightarrow n^2=...$	Solves their 3TQ in n^2 (usual rules and allow for one correct root if no working). May change variable e.g., $n^2\to x$ Ignore the labelling of roots (e.g. "n = ...") Allow for solving as a quartic if one root correct but requires $pn^4+qn^2+r=0$ oe, $p,q,r\neq 0$ **Requires previous M mark.**	**dM1**
	$n^2=9\Rightarrow n=3$	n = 3 and no other unrejected solutions. $n=\pm 3$ is A0 Must follow a correct equation.	**A1**
			(4)
			Total 8

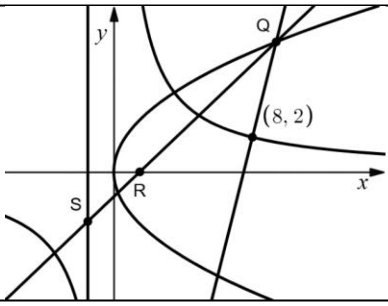
Question Number	Scheme	Notes	Marks
8	$f(k) = 7^{k-1} + 8^{2k+1}$		
	General guidance: Apply the Way that best fits the overall approach. Condone work in e.g., *n* instead of *k*. Allow use of –57 but if any different multiples of 57 are involved, e.g., 114, the last A1 additionally requires "114 is a multiple of/divisible by (but not "factor of") 57" oe for each case. Ignore work re the divisibility of f(2), f(3) etc but starting with e.g., f(2) scores a max of 01110. **Final A1**: There must be evidence that true for $n = k \Rightarrow$ true for $n = k+1$ but it could be minimal and be scored in a conclusion or a narrative or via both. So if e.g., "Assume true for $n = k$ ..." is seen in the work followed by "true for $n = k+1$" in a conclusion this is sufficient. May say "is divisible by 57" for "true". Condone "for all $n \in \mathbb{Z}$" but **not** $n \in \mathbb{R}$		
Way 1 $f(k+1) - f(k)$	$n = 1$: $f(1) = \left[7^0 + 8^3 =\right] 513$, $513 \div 57 = 9$ oe	Obtains 513 for f(1) and **shows** 513 is divisible by 57. Allow $\frac{1+512}{57} = 9$	**B1**
	$[f(k+1) =]7^{(k+1)-1} + 8^{2(k+1)+1} \left\{= 7^k + 8^{2k+3}\right\}$	Attempts $f(k+1)$	**M1**
	$[f(k+1) - f(k) =]$ $7\left(7^{k-1}\right) - 7^{k-1} + 8^2\left(8^{2k+1}\right) - 8^{2k+1}$	Obtains expression for $f(k+1) - f(k)$ in 7^{k-1} and 8^{2k+1} only	**M1**
	$= 6\left(7^{k-1} + 8^{2k+1}\right) + 57\left(8^{2k+1}\right)$ $\Rightarrow f(k+1) = 7f(k) + 57\left(8^{2k+1}\right)$ or $= 63\left(7^{k-1} + 8^{2k+1}\right) - 57\left(7^{k-1}\right)$ $\Rightarrow f(k+1) = 64f(k) - 57\left(7^{k-1}\right)$	M1: Obtains expression for $f(k+1)$ in terms of $f(k)$. May not see $f(k+1) = ...$ A1: Correct expression. **Must see** $f(k+1) = ...$ Allow if e.g., $7f(k)$ written as $7\left(7^{k-1} + 8^{2k+1}\right)$ or $7\left(7^{k-1}\right) + 7\left(8^{2k+1}\right)$	**M1** **A1**
	Shown **true for *n* = 1** and if **true for *n* = *k* <u>then</u> true for *n* = *k* + 1** so **true for all *n*** $(\in \mathbb{Z}^+)$	Makes correct conclusion or narrative with no errors throughout. Minimum in **bold**. Requires all previous marks but can follow B0 if that mark was withheld for omitting to show that 513 is divisible by 57.	**A1**
			(6)
Way 2 $f(k+1) =$	$n = 1$: $f(1) = \left[7^0 + 8^3 =\right] 513$, $513 \div 57 = 9$ oe	Obtains 513 for f(1) and **shows** 513 is divisible by 57. Allow $\frac{1+512}{57} = 9$	**B1**
	$[f(k+1) =]7^{(k+1)-1} + 8^{2(k+1)+1} \left\{= 7^k + 8^{2k+3}\right\}$	Attempts $f(k+1)$	**M1**
	$\left[f(k+1) =\right] 7\left(7^{k-1}\right) + 8^2\left(8^{2k+1}\right)$	Obtains expression for $f(k+1)$ in 7^{k-1} and 8^{2k+1} only	**M1**
	$= 7\left(7^{k-1} + 8^{2k+1}\right) + 57\left(8^{2k+1}\right)$ $\Rightarrow f(k+1) = 7f(k) + 57\left(8^{2k+1}\right)$ or $= 64\left(7^{k-1} + 8^{2k+1}\right) - 57\left(7^{k-1}\right)$ $\Rightarrow f(k+1) = 64f(k) - 57\left(7^{k-1}\right)$	M1: Obtains expression for $f(k+1)$ in terms of $f(k)$. May not see $f(k+1) = ...$ A1: Correct expression. **Must see** $f(k+1) = ...$ Allow if e.g., $7f(k)$ written as $7\left(7^{k-1} + 8^{2k+1}\right)$ or $7\left(7^{k-1}\right) + 7\left(8^{2k+1}\right)$	**M1** **A1**
	Shown **true for *n* = 1** and if **true for *n* = *k* <u>then</u> true for *n* = *k* + 1** so **true for all *n*** $(\in \mathbb{Z}^+)$	Makes correct conclusion or narrative with no errors throughout. Minimum in **bold**. Requires all previous marks but can follow B0 if that mark was withheld for omitting to show that 513 is divisible by 57.	**A1**
			(6)

Question Number	Scheme	Notes	Marks
8 **Way 3** $f(k+1)-mf(k)$	$n=1:\quad f(1)=\left[7^0+8^3=\right]513$, $513\div 57=9$ oe	Obtains 513 for f(1) and **shows** 513 is divisible by 57. Allow $\frac{1+512}{57}=9$	**B1**
	$\left[f(k+1)=\right]7^{(k+1)-1}+8^{2(k+1)+1}\left\{=7^k+8^{2k+3}\right\}$	Attempts $f(k+1)$	**M1**
	$f(k+1)-mf(k)$ $=7\left(7^{k-1}\right)-\left(7^{k-1}\right)m+8^2\left(8^{2k+1}\right)-\left(8^{2k+1}\right)m$	Obtains expression for $f(k+1)-mf(k)$ in 7^{k-1} and 8^{2k+1} only	**M1**
	e.g., $m=7\Rightarrow$ $f(k+1)-7f(k)=57\left(8^{2k+1}\right)$ $\Rightarrow f\left(k+1\right)=7f(k)+57\left(8^{2k+1}\right)$ e.g., $m=64\Rightarrow$ $f(k+1)-64f(k)=-57\left(7^{k-1}\right)$ $\Rightarrow f\left(k+1\right)=64f(k)-57\left(7^{k-1}\right)$	M1: Obtains expression for $f(k+1)$ in terms of $f(k)$ using a value for m. May not see $f(k+1)=...$ A1: A correct expression. **Must see** $f(k+1)=...$ Allow if $\beta f(k)$ written as $\beta\left(7^{k-1}+8^{2k+1}\right)$ or $\beta\left(7^{k-1}\right)+\beta\left(8^{2k+1}\right)$	**M1** **A1**
	Shown **true for *n* = 1** and if **true for *n* = *k*** <u>**then**</u> **true for *n* = *k* + 1** so **true for all *n*** $(\in\mathbb{Z}^+)$	Makes correct conclusion or narrative with no errors throughout. Minimum in **bold**. Requires all previous marks but can follow B0 if that mark was withheld for omitting to show that 513 is divisible by 57.	**A1**
			(6)
Way 4 $f(k)=57\lambda$	$n=1:\quad f(1)=\left[7^0+8^3=\right]513$, $513\div 57=9$ oe	Obtains 513 for f(1) and **shows** 513 is divisible by 57. Allow $\frac{1+512}{57}=9$	**B1**
	$\left[f(k+1)=\right]7^{(k+1)-1}+8^{2(k+1)+1}\left\{=7^k+8^{2k+3}\right\}$	Attempts $f(k+1)$	**M1**
	$\left[f\left(k+1\right)=\right]7\left(7^{k-1}\right)+8^2\left(8^{2k+1}\right)$	Obtains expression for $f(k+1)$ in 7^{k-1} and 8^{2k+1} only	**M1**
	$=7\left(7^{k-1}+8^{2k+1}\right)+57\left(8^{2k+1}\right)$ $f\left(k\right)=57\lambda\Rightarrow f\left(k+1\right)=399\lambda+57\left(8^{2k+1}\right)$ or $=7\times 57\lambda+57\left(8^{2k+1}\right)$ or $=64\left(7^{k-1}+8^{2k+1}\right)-57\left(7^{k-1}\right)$ $f\left(k\right)=57\lambda\Rightarrow f\left(k+1\right)=64\times 57\lambda-57\left(7^{k-1}\right)$ or $=3648\lambda-57\left(7^{k-1}\right)$	M1: Obtains expression for $f(k+1)$ in terms of λ with $f\left(k\right)=57\lambda$ seen. May not see $f(k+1)=...$ A1: Correct expression **Must see** $f(k+1)=...$	**M1** **A1**
	Shown **true for *n* = 1** and if **true for *n* = *k*** <u>**then**</u> **true for *n* = *k* + 1** so **true for all *n*** $(\in\mathbb{Z}^+)$	Makes correct conclusion or narrative with no errors throughout. Minimum in **bold**. Requires all previous marks but can follow B0 if that mark was withheld for omitting to show that 513 is divisible by 57.	**A1**
			(6)
			Total 6

Question Number	Scheme	Notes	Marks
9(a)	$y=c^2x^{-1}$ $\quad \frac{dy}{dx}=-c^2x^{-2}=-\frac{c^2}{x^2}$ $\quad \left(ct,\frac{c}{t}\right)\Rightarrow\frac{dy}{dx}=-\frac{c^2}{c^2t^2}$ $xy=c^2$ $\quad y+x\frac{dy}{dx}=0$ $\quad \frac{dy}{dx}=-\frac{y}{x}\Rightarrow\frac{-\frac{c}{t}}{ct}$ $x=ct \quad y=\frac{c}{t}$ $\quad \frac{dx}{dt}=c \quad \frac{dy}{dt}=-ct^{-2}$ $\quad \frac{dy}{dx}=-\frac{ct^{-2}}{c}$ Correct expression for $\frac{dy}{dx}$ in terms of *c* and *t* (or just *t*). Award when seen and isw. Allow for a correct $\frac{dx}{dy}$ or $-\frac{dx}{dy}$		**B1**
	$m_T=-\frac{1}{t^2}\Rightarrow m_N=t^2$	Correct perpendicular gradient rule for their $\frac{dy}{dx}$ in terms of *t* (or *c* and *t*)	**M1**
	$y-\frac{c}{t}="t^2"(x-ct)$ **or** $y="t^2"x+b\Rightarrow\frac{c}{t}="t^2"(ct)+b\Rightarrow b=...$	Correct straight line method with a changed gradient in terms of *t* (or *c* and *t*) with coordinates correctly placed. Condone the use of $y=mx+c$ instead of e.g. $y=mx+b$	**M1**
	$ty-c=t^3x-ct^4$ **or** $y=t^2x+\frac{c}{t}-ct^3$ $\Rightarrow t^3x-ty=c\left(t^4-1\right)$*	Fully correct proof with at least one intermediate line before printed answer but allow if equation reversed and/or order altered e.g., $\left(-1+t^4\right)c=-ty+t^3x$	**A1***
	Score a maximum of 0110 if they start with just $\frac{dy}{dx}=-\frac{1}{t^2}$ and 0010 if just $m_N=t^2$		
			(4)

Question Number	Scheme	Notes	Marks
9(b)	$(8,\ 2) \Rightarrow$ e.g., $c^2 = 16$, $\mathbf{c = 4}$; $ct = 8$ or $\frac{c}{t} = 2 \Rightarrow \mathbf{t = 2}$	Correct values for c and t seen, used or implied (e.g., by correct normal). If $c = \pm 4,\ t = \pm 2$ then the positive values must be implied by subsequent work	**B1**
	Note that another way of finding t is by using c = 4 and (8, 2) in the normal: $\Rightarrow 8t^3 - 2t = 4\left(t^4 - 1\right) \Rightarrow 4t^4 - 8t^3 + 2t - 4 = (t-2)\left(4t^3 + 2\right) = 0 \Rightarrow t = 2$		
	normal : $8x - 2y = 60 \Rightarrow$ $y = 4x - 30$ or $x = \frac{15}{2} + \frac{1}{4}y$ $\Rightarrow (4x-30)^2 = 6x$ or $y^2 = 45 + \frac{3}{2}y$	Uses their values of c and t in the given normal $t^3x - ty = c\left(t^4 - 1\right)$ [could repeat the work in (a) with $y = 16x^{-1}$] and substitutes into the parabola to obtain a quadratic equation. **Note that appropriate work must be seen for this mark**. $4x - 30 = \sqrt{6x}$ must be followed by a credible attempt to square (i.e., a 3TQ on LHS and ...x on the RHS) but see note below	**M1**
	Note that replacing x with e.g., k^2 in $4x - 30 = \sqrt{6x} \rightarrow$ $4k^2 - 30 = \sqrt{6}\,k \Rightarrow k = \frac{\sqrt{6} \pm \sqrt{6 - 4(4)(-30)}}{2(4)} = \frac{5\sqrt{6}}{4},\ -\sqrt{6} \Rightarrow x = \frac{75}{8},\ 6$ Scores the M1 for the quadratic in k and the dM1 for solving via usual rules and also reaching $x = \ldots$ by squaring.		
	$16x^2 - 246x + 900 = 0 \Rightarrow 8x^2 - 123x + 450 = 0$ $\Rightarrow (8x-75)(x-6) = 0 \Rightarrow x = \ldots$ or $2y^2 - 3y - 90 = 0 \Rightarrow (2y-15)(y+6) = 0 \Rightarrow y = \ldots$	Solves 3TQ (usual rules – one correct root if no working). **Requires previous method mark.**	**dM1**
	$x = \frac{75}{8},\ y = \frac{15}{2}$ or e.g., $Q(9.375,\ 7.5)$	Correct values/coordinates. Allow any equivalent fractions. If a second point is given e.g., (6, -6) or (6, 6) score A0 if it is not rejected in (b).	**A1**
			(4)
Alt **Approaches using parametric coords**	$\mathbf{c = 4},\ \mathbf{t = 2}$	Correct values for c and t seen or used	**B1**
	Let Q have coordinates $\left(\frac{3}{2}k^2, 3k\right)$: Substituting into the normal with c = 4 and t = 2: $8\left(\frac{3}{2}k^2\right) - 2(3k) = 4(16-1)$ **OR** Since gradient of normal to hyperbola $= t^2 = 4$, gradient of AQ where A is $(8,\ 2) = \frac{3k-2}{\frac{3}{2}k^2 - 8} = 4$ Forms a quadratic equation with their values. The equation in case 2 implies the B1.		**M1**
	$12k^2 - 6k = 60$ or $3k - 2 = 6k^2 - 32 \Rightarrow 6k^2 - 3k - 30 = 0$ $\Rightarrow 2k^2 - k - 10 = 0 \Rightarrow (2k-5)(k+2) = 0 \Rightarrow k = \ldots\left[\frac{5}{2}\right]$ $\Rightarrow x = \ldots$ or $y = \ldots$	Solves 3TQ (usual rules – one correct root if no working) **and** proceeds to a value of x or y **Requires previous method mark.**	**dM1**
	$x = \frac{75}{8},\ y = \frac{15}{2}$ or e.g., $Q\left(9\frac{3}{8}, 7\frac{1}{2}\right)$	Correct values/coordinates. Allow any equivalent fractions. If a second point is given e.g., (6, -6) or (6, 6) score A0 if it is not rejected in (b).	**A1**
			(4)

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Notes</th><th>Marks</th></tr>
<tr><td rowspan="6">9(c)</td><td colspan="2">$R\left(\frac{3}{2}, 0\right)$
Correct coordinates for the focus seen or used. Can score anywhere e.g., written across the question. Condone sight of $\left(0, \frac{3}{2}\right)$ if used correctly e.g. in gradient calculation. If on a diagram, accept $\frac{3}{2}$ appropriately placed. Accept 1.5, $\frac{6}{4}$ etc. for $\frac{3}{2}$. Just "a or $x = \frac{3}{2}$" or "$R = \frac{3}{2}$" is insufficient. There must be some recognition of the position of R.</td><td>B1</td></tr>
<tr><td colspan="2">Allow work with decimals for the 3 M marks.</td><td></td></tr>
<tr><td colspan="2">QR: e.g., $y - 0 = \left(\frac{"\frac{15}{2}" - 0}{"\frac{75}{8}" - "\frac{3}{2}"}\right)(x - "\tfrac{3}{2}")$ or $y = \left(\frac{"\frac{15}{2}" - 0}{"\frac{75}{8}" - "\frac{3}{2}"}\right)x + c \Rightarrow 0 = \frac{20}{21}\left(\frac{3}{2}\right) + c \Rightarrow c = ...$
Correctly forms equation of QR for their Q and R. Q could be "made up" or be an incorrect choice from part (b) but must have real coordinates (A, B), $A > 0$, $B \neq 0$ so allow e.g., (6, 6) and (6, -6). R must be of form (α, 0), $\alpha > 0$
Allow if a correct gradient is seen but wrongly calculated before line equation is given.
If using $y = mx + c$ the equation must be formed correctly and "c = ..." reached following correct placement of (α, 0).
For $0 = "\tfrac{3}{2}"m + c$, $"\tfrac{15}{2}" = "\tfrac{75}{8}"m + c \Rightarrow m = ...,\ c = ...$ must find both m and c with one correct
M0 for a vertical line or if a normal gradient is used</td><td>M1</td></tr>
<tr><td>$y = \frac{20}{21}x - \frac{10}{7}$, $x = -\frac{3}{2}$
$\Rightarrow y = \frac{20}{21}\left(-\frac{3}{2}\right) - \frac{10}{7} = -\frac{10}{7} - \frac{10}{7} = -\frac{20}{7}$</td><td>Substitutes $x = -\alpha$, $\alpha > 0$ into their equation to find a value for the y coordinate.
Must be using a consistent α
Requires previous M mark.</td><td>dM1</td></tr>
<tr><td>$S\left(-\frac{3}{2}, -\frac{20}{7}\right) \Rightarrow$
$QS = \sqrt{\left(\frac{75}{8} - \left(-\frac{3}{2}\right)\right)^2 + \left(\frac{15}{2} - \left(-\frac{20}{7}\right)\right)^2}$</td><td>Applies correct distance formula for their $Q(A, B)$, $A > 0$, $B \neq 0$ and $S(-\alpha, \pm\beta)$ $\alpha > 0$ and consistent, $\beta \neq 0$
Implied by 15.017857... otherwise working must be seen.
Requires both previous M marks.
Note that using $(6, 6)$ or $(6, -6) \rightarrow QS = \frac{25}{2}$</td><td>ddM1</td></tr>
<tr><td colspan="2">$\left[= \sqrt{\left(\frac{87}{8}\right)^2 + \left(\frac{145}{14}\right)^2} = \sqrt{\frac{7569}{64} + \frac{21025}{196}} = \sqrt{\frac{707281}{3136}} = \right] \Rightarrow QS = \frac{841}{56}$
Correct exact distance. Any exact equivalent e.g., $15\frac{1}{56}$ and may not be in simplest form</td><td>A1</td></tr>
<tr><td>Alt

For the last two marks ($QS = QR + RS$)</td><td colspan="2">$QS = QR + RS$ but QR = shortest distance of Q to directrix $= \frac{75}{8} + \frac{3}{2} = \frac{87}{8}$
$QS = \sqrt{\left(0 - \left(-\frac{20}{7}\right)\right)^2 + \left(\frac{3}{2} - \left(-\frac{3}{2}\right)\right)^2} + \frac{87}{8} = \frac{29}{7} + \frac{87}{8} = \frac{841}{56}$
M1: A full method correct for their Q and S. Implied only by awrt 15.017857...
A1: Correct exact distance (any equivalent)</td><td></td></tr>
<tr><td></td><td colspan="2"></td><td></td></tr>
<tr><td colspan="4" align="right">(5)</td></tr>
<tr><td colspan="4" align="right">Total 13</td></tr>
<tr><td colspan="4" align="right">PAPER TOTAL: 75 marks</td></tr>
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